Prueba 2 - MATE22

1. (8 points)

   Calcule: $\sum_{k=24}^{89} \left(k^3 - 8k^2 - 6k\right)$

2. (8 points)
   Calcule la siguiente suma utilizando propiedades de sumatorias:

   $22 + 33 + 50 + 73 + \ldots\ldots + 14\,857$

3. (8 points)

   Determine la siguiente suma general: $\sum_{i=1}^{n} \left(5i^2 - 15i\right)$

4. (8 points)
   Calcule la suma de todos los múltiplos de 3 que están comprendidos entre 500 y 950.

5. (8 points)

   Calcule: $\sum_{k=55}^{190} \frac{3}{(k+3)(k+2)}$

Pauta Prueba 2 - MATE22   Forma 1
Answer Section

PROBLEM

1. ANS:
   Solución

$$\sum_{k=24}^{89}\left(k^3-8k^2-6k\right)=\sum_{k=1}^{89}\left(k^3-8k^2-6k\right)-\sum_{k=1}^{23}\left(k^3-8k^2-6k\right)=$$

$$=\sum_{k=1}^{89}k^3-8\sum_{k=1}^{89}k^2-6\sum_{k=1}^{89}k-\left(\sum_{k=1}^{23}k^3-8\sum_{k=1}^{23}k^2-6\sum_{k=1}^{23}k\right)=$$

$$=\left[\frac{89\cdot 90}{2}\right]^2-8\cdot\frac{89\cdot 90\cdot 179}{6}-6\cdot\frac{89\cdot 90}{2}-\left(\left[\frac{23\cdot 24}{2}\right]^2-8\cdot\frac{23\cdot 24\cdot 47}{6}-6\cdot\frac{23\cdot 24}{2}\right)$$

$$=14\,104\,275-39\,928=14\,064\,347$$

   PTS: 8

2. ANS:
   Solución

$$22+33+50+73+\ldots\ldots+14\,857$$

$$\left.\begin{array}{l} a+b+c=22 \\ 3a+b=11 \\ 2a=6 \end{array}\right\} \Rightarrow a=3,\ b=2,\ c=17$$

$$\Rightarrow\quad a_n=3n^2+2n+17=14\,857 \qquad \Rightarrow \qquad 3n^2+2n-14840=0 \qquad \Rightarrow \qquad n=70$$

$$22+33+50+73+\ldots\ldots+14\,857=\sum_{k=1}^{70}\left(3k^2+2k+17\right)=$$

$$=3\sum_{k=1}^{70}k^2+2\sum_{k=1}^{70}k+\sum_{k=1}^{70}17=\frac{3\cdot 70\cdot 71\cdot 141}{6}+\frac{2\cdot 70\cdot 71}{2}+70\cdot 17=356\,545$$

   PTS: 8

3. ANS:
Solución

$$\sum_{i=1}^{n}\left(5i^2-15i\right)=5\sum_{i=1}^{n}i^2-15\sum_{i=1}^{n}i=$$

$$=5\cdot\frac{n(n+1)(2n+1)}{6}-15\cdot\frac{n(n+1)}{2}\cdot\frac{3}{3}=\frac{5n(n+1)(2n+1)-45n(n+1)}{6}=$$

$$=\frac{10n^3+15n^2+5n-45n^2-45n}{6}=\frac{10n^3-30n^2-40n}{6}$$

PTS: 8

4. ANS:
Solución

$$\frac{500}{3}=166.667\longrightarrow \text{Se aproxima al entero superior}=167\longrightarrow a_1=3\cdot 167=501$$

$$\frac{950}{3}=316.667\longrightarrow \text{Se aproxima al entero inferior}=316\longrightarrow a_n=3\cdot 316=948$$

Hay que sumar $501+504+507+\ldots\ldots+945+948$

=> $a_k=3k+498$

$$a_n=3n+498\longrightarrow 948=3\cdot n+498\longrightarrow n=150$$

$$\sum_{k=1}^{150}(3k+498)=3\sum_{k=1}^{150}k+\sum_{k=1}^{150}498=3\cdot\frac{150\cdot 151}{2}+150\cdot 498=33975+74700=108675$$

PTS: 8

5. ANS:
Solución

$$\frac{3}{(k+3)(k+2)}=\frac{A}{k+3}+\frac{B}{k+2}\qquad /\cdot(k+3)(k+2)$$

$$3=A(k+2)+B(k+3)$$

$$Si\ k=-3\quad =>\quad -3=A$$

$$Si\ k=-2\quad =>\quad 3=B$$

$$\sum_{k=55}^{190}\frac{3}{(k+3)(k+2)}=\sum_{k=55}^{190}\left(\frac{-3}{k+3}+\frac{3}{k+2}\right)=\frac{-3}{190+3}+\frac{3}{55+2}=\frac{-3}{193}+\frac{3}{57}=\frac{408}{11001}=\frac{136}{3667}$$

PTS: 8